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P1 Exam-Style Practice Booklet — Chapters 1-4

Cambridge A-Level Mathematics 9709 Paper 1

Name: _____

Class: _____

Date: _____

Chapter 1: Quadratics

1. The equation $x^2 + (k + 2)x + (3k + 1) = 0$, where k is a constant, has two equal roots.

(a) Show that $k^2 - 8k - 8 = 0$. [3]

(b) Hence find the possible values of k , giving your answers in the form $a \pm b\sqrt{c}$ where a , b and c are integers. [3]

(c) For the larger value of k , find the value of the repeated root. [2]

2. The function $f(x) = 2x^2 - 8x + 5$ is defined for $x \in \mathbb{R}$.

(a) Express $f(x)$ in the form $a(x + b)^2 + c$, where a , b and c are constants. [3]

(b) Hence write down the coordinates of the minimum point of the curve $y = f(x)$. [2]

(c) Find the exact solutions of the equation $f(x) = 0$, giving your answers in simplified surd form. [3]

3. The line $y = 2x + 3$ intersects the curve $y = x^2 + kx + 7$ at two distinct points, where k is a constant.

(a) Show that $x^2 + (k - 2)x + 4 = 0$. [2]

(b) Find the range of values of k for which the line intersects the curve at two distinct points. [4]

(c) Given that the line is a tangent to the curve, find the possible values of k . [2]

4. A rectangular garden has length $(x + 5)$ metres and width $(x - 2)$ metres. The area of the garden is 60 m^2 .

(a) Show that $x^2 + 3x - 70 = 0$. [2]

(b) Hence find the length and width of the garden. [3]

(c) Find the minimum possible perimeter of a rectangle with area 60 m^2 , justifying that your answer is a minimum. [4]

5. The equation $2x^2 + (p + 1)x + (p - 1) = 0$, where p is a constant, has roots α and β .

(a) Given that $\alpha + \beta = 3$, find the value of p . [3]

(b) For this value of p , find the value of $\alpha\beta$. [2]

(c) Hence form a quadratic equation whose roots are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$, giving your answer in the form $ax^2 + bx + c = 0$ where a , b and c are integers. [3]

6. Solve the inequality $2x^2 - 7x + 3 \leq 0$, giving your answer in exact form. [4]

Hence find the set of values of x for which $2x^2 - 7x + 3 \leq 0$ and $x > 1$. [2]

7. The curve C has equation $y = x^2 - 4x + 7$ and the line l has equation $y = 2x + k$, where k is a constant.

(a) Show that the x -coordinates of the points of intersection satisfy $x^2 - 6x + (7 - k) = 0$. [2]

(b) Find the range of values of k for which l intersects C at two distinct points. [3]

(c) Given that l is a tangent to C , find the coordinates of the point of tangency. [3]

8. The function $f(x) = 3x^2 + 12x + 7$.

(a) Express $f(x)$ in the form $a(x + b)^2 + c$. [3]

(b) State the minimum value of $f(x)$ and the value of x at which it occurs. [2]

(c) Solve the equation $f(x) = 7$, giving your answers in exact form. [3]

(d) Find the range of values of x for which $f(x) > 7$. [2]

Chapter 2: Functions

9. The functions f and g are defined by

$$f(x) = 3x - 2, \quad x \in \mathbb{R}$$

$$g(x) = x^2 + 1, \quad x \in \mathbb{R}$$

(a) Find $fg(x)$ and simplify your answer. [2]

(b) Find $gf(x)$ and simplify your answer. [2]

(c) Solve the equation $fg(x) = gf(x)$. [3]

10. The function f is defined by $f(x) = \frac{2x+1}{x-3}$ for $x \in \mathbb{R}$, $x \neq 3$.

(a) Find $f^{-1}(x)$, stating its domain. [4]

(b) Show that $f(f(x)) = x$ for all x in the domain of f . [3]

(c) State the relationship between the graph of $y = f(x)$ and the graph of $y = f^{-1}(x)$. [1]

11. The function f is defined by $f(x) = x^2 - 6x + 5$ for $x \geq 3$.

(a) Express $f(x)$ in the form $(x - a)^2 + b$. [2]

(b) State the range of f . [2]

(c) Find $f^{-1}(x)$, stating its domain and range. [4]

12. The curve $y = f(x)$ passes through the point $(2, 5)$. The curve $y = g(x)$ is obtained from $y = f(x)$ by the transformation $(x, y) \rightarrow (x + 3, 2y)$.

(a) Find the coordinates of the image of the point $(2, 5)$ on $y = g(x)$. [2]

(b) Express $g(x)$ in terms of f . [2]

(c) The curve $y = f(x)$ has a minimum point at $(0, 1)$. Find the coordinates of the minimum point of $y = g(x)$. [2]

(d) Describe fully the transformation that maps the graph of $y = f(x)$ onto the graph of $y = g(x)$. [2]

13. The function f is defined by $f(x) = 2x^2 - 8x + 3$ for $x \in \mathbb{R}$.

(a) Find the coordinates of the vertex of the graph of $y = f(x)$. [3]

(b) The graph of $y = f(x)$ is translated by the vector $\begin{pmatrix} 2 \\ -1 \end{pmatrix}$ to give the graph of $y = g(x)$. Find $g(x)$ in the form $ax^2 + bx + c$. [3]

(c) The graph of $y = f(x)$ is reflected in the y -axis to give the graph of $y = h(x)$. Find $h(x)$. [2]

14. The function f is defined by $f(x) = \sqrt{x - 2}$ for $x \geq 2$.

(a) State the range of f . [1]

(b) Find $f^{-1}(x)$, stating its domain. [3]

(c) On the same axes, sketch the graphs of $y = f(x)$ and $y = f^{-1}(x)$, stating the equation of the line of symmetry. [3]

(d) Solve the equation $f(x) = f^{-1}(x)$. [3]

Chapter 3: Coordinate Geometry

15. The points $A(1, 2)$ and $B(7, 10)$ are given.

- (a) Find the length of AB , giving your answer in simplified surd form. [2]
- (b) Find the coordinates of the midpoint M of AB . [2]
- (c) Find the equation of the perpendicular bisector of AB , giving your answer in the form $ax + by + c = 0$. [4]

16. The line l_1 has equation $3x + 4y - 12 = 0$ and the line l_2 is perpendicular to l_1 , passing through the point $(6, -1)$.

- (a) Find the gradient of l_1 . [1]
- (b) Find the equation of l_2 in the form $y = mx + c$. [3]
- (c) Find the coordinates of the point of intersection of l_1 and l_2 . [3]
- (d) Find the perpendicular distance from the origin to l_1 . [3]

17. The circle C has equation $x^2 + y^2 - 6x + 8y - 11 = 0$.

- (a) Find the centre and radius of C . [3]
- (b) Show that the point $P(7, 0)$ lies on the circle. [1]
- (c) Find the equation of the tangent to the circle at P , giving your answer in the form $ax + by + c = 0$. [4]

18. The circle C has centre $(3, -2)$ and passes through the point $(7, 1)$.

- (a) Find the radius of C . [2]
- (b) Find the equation of C in the form $(x - a)^2 + (y - b)^2 = r^2$. [2]
- (c) The line $y = 2x - 1$ intersects C at points A and B . Find the coordinates of A and B . [5]
- (d) Find the length of the chord AB , giving your answer in simplified surd form. [2]

19. The points $A(-2, 1)$, $B(4, 5)$ and $C(6, -1)$ form a triangle.

- (a) Show that triangle ABC is isosceles. [3]
- (b) Find the area of triangle ABC . [4]
- (c) Find the equation of the circle with diameter AB , giving your answer in the form $x^2 + y^2 + ax + by + c = 0$. [4]

20. The line l has equation $y = x + k$ and the circle C has equation $x^2 + y^2 = 25$.

- (a) Show that the x -coordinates of the points of intersection satisfy $2x^2 + 2kx + (k^2 - 25) = 0$. [2]
- (b) Find the range of values of k for which l intersects C at two distinct points. [3]
- (c) Given that l is a tangent to C , find the possible values of k , giving your answers in simplified surd form. [3]
- (d) For the positive value of k , find the coordinates of the point of tangency. [3]

Chapter 4: Circular Measure

21. A sector of a circle has radius 8 cm and angle $\frac{3\pi}{5}$ radians.

(a) Find the length of the arc of the sector, giving your answer in terms of π . [2]

(b) Find the area of the sector, giving your answer in terms of π . [2]

(c) Find the perimeter of the sector, giving your answer in terms of π . [2]

22. The diagram shows a sector OAB of a circle with centre O and radius r cm. The angle AOB is θ radians. The perimeter of the sector is 20 cm.

(a) Show that the area A cm² of the sector is given by $A = 10r - r^2$. [4]

(b) Find the maximum area of the sector, justifying that your answer is a maximum. [4]

(c) Find the value of θ for which this maximum area occurs. [2]

23. A circle has radius 6 cm. A chord AB subtends an angle of $\frac{2\pi}{3}$ radians at the centre O .

(a) Find the length of the chord AB , giving your answer in simplified surd form. [3]

(b) Find the area of the sector OAB , giving your answer in terms of π . [2]

(c) Find the area of the triangle OAB , giving your answer in simplified surd form. [3]

(d) Hence find the area of the minor segment cut off by the chord AB , giving your answer in the form $a\pi - b\sqrt{3}$ where a and b are integers. [2]

24. The diagram shows a semicircle with diameter $AB = 12$ cm. A point C lies on the semicircle such that the angle $CAB = \frac{\pi}{6}$ radians.

(a) Show that $AC = 6\sqrt{3}$ cm. [3]

(b) Find the area of the sector OAC , where O is the midpoint of AB , giving your answer in terms of π . [3]

(c) Find the area of the region bounded by the arc AC , the line CB and the line AB . [4]

25. Two circles, each of radius 5 cm, have centres O_1 and O_2 such that $O_1O_2 = 8$ cm. The circles intersect at points P and Q .

(a) Show that the angle $PO_1Q = 2 \arccos\left(\frac{4}{5}\right)$ radians. [4]

(b) Find the area of the sector O_1PQ , giving your answer in terms of $\arccos\left(\frac{4}{5}\right)$. [3]

(c) Find the area of the triangle O_1PQ , giving your answer in simplified surd form. [3]

(d) Hence find the area of the region common to both circles, giving your answer in terms of $\arccos\left(\frac{4}{5}\right)$ and surds. [4]