

Contents

A-Level Mathematics 9709 — Topic 1: Quadratics	1
Monthly Exam Revision Notes	1
Key Knowledge Points	1
Worked Examples	3
Practice Checklist	6

A-Level Mathematics 9709 — Topic 1: Quadratics

Monthly Exam Revision Notes

Key Knowledge Points

1. Solving Quadratic Equations

A quadratic equation has the form:

$$ax^2 + bx + c = 0, \quad a \neq 0$$

Three main methods:

- **Factorisation**
 - Find two numbers that multiply to ac and add to b
 - Split the middle term, then factor by grouping
- **Completing the Square**
 - Write in the form $(x + p)^2 + q = 0$
 -

$$x^2 + bx + c = \left(x + \frac{b}{2}\right)^2 - \frac{b^2}{4} + c$$

- **Quadratic Formula**
 -

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

2. Discriminant and Nature of Roots

For $ax^2 + bx + c = 0$, the discriminant is:

$$\Delta = b^2 - 4ac$$

Condition	Nature of Roots
$\Delta > 0$	Two distinct real roots
$\Delta = 0$	One repeated real root (tangent)
$\Delta < 0$	No real roots

3. Maximum / Minimum and Line of Symmetry

For $y = ax^2 + bx + c$:

- Axis of symmetry:

$$x = -\frac{b}{2a}$$

- Vertex (turning point):

$$\left(-\frac{b}{2a}, c - \frac{b^2}{4a}\right)$$

- If $a > 0$: minimum point, opens upward
- If $a < 0$: maximum point, opens downward

Completed square form: $y = a(x - h)^2 + k$ gives vertex (h, k) .

4. Quadratic Graphs Checklist

- Shape: parabola (upward if $a > 0$, downward if $a < 0$)
 - y -intercept: set $x = 0$, gives $y = c$
 - x -intercepts: solve $ax^2 + bx + c = 0$
 - Vertex coordinates
 - Axis of symmetry
 - Discriminant to check number of x -intercepts
-

5. Simultaneous Equations (One Linear, One Quadratic)

- Rearrange the linear equation to express one variable in terms of the other
 - Substitute into the quadratic equation
 - Solve the resulting quadratic
 - Back-substitute to find the other variable
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6. Intersection of a Line and a Quadratic Curve

- Substitute the line equation into the curve equation
- The resulting quadratic's discriminant determines the number of intersections:

Discriminant	Intersections
$\Delta > 0$	Two points
$\Delta = 0$	One point (tangent)
$\Delta < 0$	No intersection

7. Disguised Quadratic Equations

Equations that become quadratic after substitution.

- Example forms:

—

$$x^4 - 5x^2 + 4 = 0 \quad \text{let } u = x^2$$

—

$$2^{2x} - 5 \cdot 2^x + 4 = 0 \quad \text{let } u = 2^x$$

—

$$x + \sqrt{x} - 6 = 0 \quad \text{let } u = \sqrt{x}$$

Always back-substitute to find the original variable.

8. Solving Quadratic Inequalities

- Solve the corresponding equation $ax^2 + bx + c = 0$
- Sketch the parabola to determine the solution region
- For $ax^2 + bx + c > 0$ with $a > 0$: solution is $x < \alpha$ or $x > \beta$
- For $ax^2 + bx + c < 0$ with $a > 0$: solution is $\alpha < x < \beta$
- For systems: find the intersection of all solution sets

Worked Examples

Example 1 — Solving by Completing the Square

Solve $x^2 - 6x + 4 = 0$ by completing the square.

$$x^2 - 6x + 4 = 0$$

$$(x - 3)^2 - 9 + 4 = 0$$

$$(x - 3)^2 = 5$$

$$x = 3 \pm \sqrt{5}$$

$x = 3 + \sqrt{5} \quad \text{or} \quad x = 3 - \sqrt{5}$

Example 2 — Quadratic Formula

Solve $2x^2 + 5x - 3 = 0$ using the quadratic formula.

$$x = \frac{-5 \pm \sqrt{25 + 24}}{4} = \frac{-5 \pm 7}{4}$$

$$x = \frac{1}{2} \text{ or } x = -3$$

Example 3 — Discriminant

Find the values of k for which $x^2 + kx + 9 = 0$ has two distinct real roots.

$$\Delta = k^2 - 36 > 0$$

$$k^2 > 36$$

$$k < -6 \text{ or } k > 6$$

Example 4 — Maximum / Minimum

Express $y = 2x^2 - 8x + 5$ in completed square form and state the minimum point.

$$y = 2(x^2 - 4x) + 5 = 2[(x - 2)^2 - 4] + 5$$

$$y = 2(x - 2)^2 - 3$$

$$\text{Minimum point: } (2, -3)$$

Example 5 — Quadratic Graph

Sketch $y = x^2 - 4x + 3$, showing all key features.

$$y = x^2 - 4x + 3$$

$$\text{Vertex } (2, -1), \text{ } x\text{-intercepts } 1, 3, \text{ } y\text{-intercept } 3$$

Example 6 — Simultaneous Equations

Solve: $y = x + 1$ and $y = x^2 - 2x + 3$

$$x + 1 = x^2 - 2x + 3$$

$$x^2 - 3x + 2 = 0$$

$$(x - 1)(x - 2) = 0$$

$$x = 1 \Rightarrow y = 2; \quad x = 2 \Rightarrow y = 3$$

$$\boxed{(1, 2) \text{ and } (2, 3)}$$

Example 7 — Line and Curve Intersection

Find the values of c for which $y = 2x + c$ is tangent to $y = x^2 + 4x + 7$.

$$x^2 + 4x + 7 = 2x + c$$

$$x^2 + 2x + (7 - c) = 0$$

Tangent: $\Delta = 0$

$$4 - 4(7 - c) = 0 \Rightarrow 4 - 28 + 4c = 0$$

$$\boxed{c = 6}$$

Example 8 — Disguised Quadratic

Solve $x^4 - 13x^2 + 36 = 0$.

Let $u = x^2$:

$$u^2 - 13u + 36 = 0 \Rightarrow (u - 4)(u - 9) = 0$$

$$x^2 = 4 \quad \text{or} \quad x^2 = 9$$

$$x = \pm 2 \text{ or } x = \pm 3$$

Example 9 — Quadratic Inequality

Solve $x^2 - 5x + 6 < 0$.

$$(x - 2)(x - 3) < 0$$

Parabola opens upward, roots at $x = 2, 3$.

$$2 < x < 3$$

Example 10 — System of Inequalities

Solve simultaneously: $x^2 - 4 < 0$ and $x^2 - 3x \geq 0$.

First: $-2 < x < 2$

Second: $x(x - 3) \geq 0 \Rightarrow x \leq 0$ or $x \geq 3$

Intersection:

$$-2 < x \leq 0$$

Practice Checklist

- ☐ Can I solve a quadratic by factorisation, completing the square, and the formula?
- ☐ Can I compute the discriminant and interpret its sign?
- ☐ Can I find the vertex and axis of symmetry from completed square form?
- ☐ Can I sketch a quadratic graph with all key features labeled?
- ☐ Can I solve a linear-quadratic simultaneous system?
- ☐ Can I use the discriminant to determine tangency conditions?
- ☐ Can I recognise and solve disguised quadratics using substitution?
- ☐ Can I solve quadratic inequalities and combine them into systems?
- ☐ Can I handle cases where $a < 0$ (downward parabola)?
- ☐ Can I check my answers by substitution?